Physics Handout 3

Astronomy & Astrophysics Olympiad Problems

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Problem 1: Keplerian Orbit

A planet moves around a star of mass M in an elliptical orbit with semi-major axis a and eccentricity e.
Tasks:

1. Derive expressions for the perihelion and aphelion velocities.
2. Find the ratio v_p/v_a.

Problem 2: Escape Velocity

A spherical asteroid of uniform density ρ has radius R.
Tasks:

1. Derive the escape velocity from its surface.
2. For what radius R does the escape velocity equal 1 m s^{-1} if $\rho = 3000$ kg m^{-3}?

Problem 3: Binary Star System

Two stars of masses M and $2M$ orbit their common center of mass in circular orbits separated by distance d.
Tasks:

1. Find the distance of each star from the center of mass.
2. Find the orbital period of the system.

Problem 4: Apparent Brightness

Two stars have absolute luminosities L_1 and $L_2 = 9L_1$. Star 1 is at distance d and star 2 is at distance $3d$.
Task: Compare their apparent brightnesses as seen from Earth.

Problem 5: Stellar Temperature

Assuming stars behave as blackbodies, show that the ratio of radii of two stars is

$$\frac{R_1}{R_2} = \sqrt{\frac{L_1}{L_2}} \left(\frac{T_2}{T_1}\right)^2$$

where L is luminosity and T is surface temperature.

Problem 6: Tidal Forces

A satellite of mass m orbits a planet of mass M at distance r.
Tasks:

1. Derive the expression for the tidal acceleration across the satellite.
2. Explain qualitatively why tidal forces scale as $1/r^3$.

Problem 7: Hohmann Transfer

Two circular coplanar orbits of radii r_1 and r_2 surround a star of mass M.
Tasks:

1. Find the total Δv required for a Hohmann transfer.
2. Determine whether transferring inward or outward requires more energy.

Problem 8: Parallax

A nearby star shows a parallax angle of $0.05''$.
Tasks:

1. Find its distance in parsecs.
2. If its apparent magnitude is $m = 6$, find its absolute magnitude.

Problem 9: Roche Limit

Derive the Roche limit for a fluid satellite of density ρ_s orbiting a planet of density ρ_p.
Task: Explain physically why solid bodies can survive closer.

Problem 10: Doppler Shift

A galaxy is observed to have a redshift $z = 0.02$.
Tasks:

1. Find its recessional velocity assuming non-relativistic motion.
2. Estimate its distance using $H_0 = 70$ km s^{-1}Mpc^{-1}.

Problem 11: Main Sequence Lifetime

Assume a star's luminosity scales as $L \propto M^{3.5}$.
Tasks:

1. Derive how main-sequence lifetime depends on mass.
2. Estimate how much shorter the Sun's lifetime would be if its mass were doubled.

Problem 12: Gravitational Potential

Derive the gravitational potential inside a uniform solid sphere of radius R.
Task: Sketch the potential as a function of distance from the center.

Problem 13: Angular Momentum Conservation

A collapsing gas cloud contracts from radius R to $R/100$.
Tasks:

1. Assuming angular momentum conservation, by what factor does its angular speed increase?
2. Explain the relevance to star formation.

Problem 14: Radiation Pressure

Derive the expression for radiation pressure exerted by sunlight on a perfectly absorbing surface.
Task: Estimate its value at Earth's orbit.

Problem 15: Transit Method

A planet transits its star causing a 1% dip in observed brightness.
Tasks:

1. Find the ratio of planetary radius to stellar radius.
2. Explain how transit duration gives orbital information.

End of Problem Set